<u>Sample relay races with an analysis at the end</u>.

R1-1. The area of a triangle is K. The altitude is first increased by 20%. The new altitude is then decreased by 20%. The value of K has been decreased by X%. Find X.

R1-2. Let T = TNYWR. If $|x-T| < 3$, find the largest possible integer value for x.

R1-3. Let T = TNYWR. Consider the sequence 1, 2, 3, ... , 45T, where 45T stands for four hundred and fifty plus the number T. Clearly, T is a digit from 0 to 9. How many multiples of 4 are in the sequence?

R2-1. Find the area enclosed by $x \geq 1$, $y \geq 1$, and $x + y \leq 4$.

R2-2. Let T = TNYWR. When 5^{T^2} is written out, how many digits are there?

R2-3. Let T = TNYWR. The areas of the faces of a box are 16, 16, 9, 9, T^2, and T^2. Find the volume of the box.

R3-1. How many positive factors does $35^2 - 32^2$ have?

R3-2. Let T = TNYWR. Find the area bounded by the positive x-axis, the positive y-axis, and the line $x + 2y = 2 \cdot T$.

R3-3. Let T = TNYWR. Given the system below, find the sum of $x + y$:

$$2x + y = T / 2$$
$$x + 2y = 2T$$

R4-1. If $f(x)=3x+2$ and $f^{10}(x)=ax+b$ where $f^2(x)=f(f(x))$, $f^3(x)=f(f(f(x)))$, etc. , find the value of b.

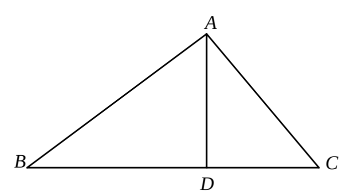
R4-2. Let T = TNYWR. Let $f(x)=ax+b$, $a\neq 0$. If $f(1)=T$ and $f(2)=2\cdot T$, compute the value of b.

R4-3. Let T = TNYWR. Let $f(x)=\dfrac{1}{2-\dfrac{1}{x-T}}$. Find the sum of all those real numbers that are not in the domain of f.

R5-1. Let a and b be the solutions to $x^2-4x+2=0$. Find the value of $(ab)(a+b)$.

R5-2. Let T = TNYWR. How many real solutions does $x^2-T\cdot x+T=0$ have?

R5-3. Let T = TNYWR. $\overline{AD}\perp\overline{BC}$, $AB=34$, $AC=20$, and $AD=3\cdot T+10$. Compute the length of $\overline{BC}$.



R6-1. Let 4 and –3 be solutions to the quadratic equation $x^2+bx+c=0$. Find the value of c.

R6-2. Let T = TNYWR. Find the slope of a line perpendicular to the line containing the points $A(-6, 0)$ and $B(18, T)$.

R6-3. Let T = TNYWR. Find the smallest possible value of x satisfying $2x+T\leq 5x+1$.

Analysis:

In general, a good way for a person to practice relays is to work them backwards. That is to say, start with the third problem and see what can be done, then go to the second problem, and finally to the first problem. In analyzing these problems I'll be writing from the standpoint of what a person could do before receiving an answer.

R1-1. The area has been changed by $\frac{4}{5}\cdot\frac{6}{5}=\frac{24}{25}=\frac{96}{100}$ so there has been a 4% decrease in the area. Pass back 4.

R1-2. Solve, obtaining $-3+T<x<T+3$. If T is an integer the answer will be $T+2$. Since T = 4, the answer to be passed back is 6.

R1-3. Dividing 450 or 451 by 4 gives 112 plus a remainder. Dividing 452, 453, 454, or 455 by 4 gives 113 plus a remainder of 0, 1, 2, or 3. Dividing 456, 457, 458, or 459 by 4 gives 114 plus a remainder. So without receiving an answer the third person can know that the answer is 112, 113, or 114. If the third person has not received the second person's answer at the three-minute mark, the third person can guess one of the three. If the third person has received an answer near the 6 minute mark, the person can keep to the 3 minute answer or change based on the value of T. And if no answer is received by the 6 minute mark the third person can guess another one of the three but this might not be that fruitful. Here, since $T=6$, the answer is 114.

R2-1. The region is a right triangle with vertex points (1,1), (1,3), and (3,1), the area is $\frac{1}{2}\cdot 2\cdot 2=2$.

R2-2. At first thought it would seem that T would be a non-negative integer. Thus, we need only consider $5^0=1,\ 5^{1^2}=5,\ 5^{2^2}=5^4=625,$ or $5^{3^2}=5^9=1953125.$ Given the time constraints T just can't exceed 3. So it would appear that the answer would have to be either 1, 3, or 7 and 7 seems unlikely given the time constraints. But then the student might think that T could equal $\sqrt{2},\sqrt{3},\sqrt{5},\ldots,\sqrt{8}$, making for the answers 2, 3 again, 4, 5, 5, and 6, so we have all the integers from 1 to 7 as possibilities. Since T = 2, the answer is 3.

R2-3. Sometimes the most general solution is the easiest. Let the edges of the box be *a*, *b*, and *c*. Then the areas of the faces are *ab*, *ab*, *bc*, *bc*, *ac*, and *ac*. Picking one of each different face and multiplying them together gives $(ab)(bc)(ac)=a^2b^2c^2$ $=V^2$. So the volume equals $\sqrt{16\cdot 9\cdot T^2}=12T$. Since T = 3, the answer is 36.

R3-1. $35^2 - 32^2 = (35+32)(35-32) = 67 \cdot 3$. Since both 67 and 3 are prime, the number has 4 factors.

R3-2. The region is a triangle with vertices (0,0), $(2T,0)$, and $(0,T)$, so the area is $\frac{1}{2} \cdot 2T \cdot T = T^2$. Since $T = 4$, the answer is 16.

R3-3. Some students will try to find $x + y$ by solving for *x* and then *y* or vice-versa. But the coefficients in this problem are such that if we add the equations, obtain $3x + 3y = \frac{5T}{2}$, and then divide by 3 getting $x + y = \frac{5T}{6}$, we can then plug in 16 to obtain the answer of $\frac{40}{3}$. Note that if the student turns in $\frac{80}{6}$, that team of three gets no points.

R4-1. By repeated composition the student may discover that there is a formula, namely, $f^n(x) = 3^n x + (3^n - 1)$. Thus, $b = 3^{10} - 1 = 59048$. The student could pass back either answer, but not both on the same paper.

R4-2. By substitution we have $T = a + b$ and $2T = 2a + b$. Subtracting gives $a = T$ and substituting gives $b = 0$ regardless of the value of *a*. In practice I've found that some but not all students recognize that they don't need the answer from #1.

R4-3. First, $x \neq T$. Simplifying gives $\frac{x - T}{2x - (2T+1)}$ so $x \neq \frac{2T+1}{2}$. Adding those two values gives a sum of $\frac{4T+1}{2}$. Since $T = 0$, the answer is $\frac{1}{2}$.

R5-1. Using the relationship between the roots and coefficients of a quadratic we have $ab = 2$ and $-(a+b) = -4$ so the product is $2 \cdot 4 = 8$.

R5-2. The discriminant is $T^2 - 4T$. It is positive for $T < 0$ or $T > 4$, giving 2 solutions, it is 0 for $T = 0$ or 4, giving 1 solution, and it is negative for $0 < T < 4$, giving no real solutions. Since $T = 8$, the answer is 2.

R5-3. Looking at the numbers a student could guess that Pythagorean triples are involved, namely the 8-15-17 (here 16-30-34) and the 3-4-5 (here 12-16-20). Letting *AD* = 16, gives *BD* = 30 and *DC* = 12, making for an answer of 42.

R6-1. Using the roots and coefficients of a quadratic makes $c = -12$. Or one could substitute, obtaining $16 + 4b + c = 0$ and $9 - 3b + c = 0$. Solving gives $b = -1$ and $c = -12$.

R6-2. The slope of $\overline{AB} = \frac{T - 0}{18 - (-6)} = \frac{T}{24}$. Since $T = -12$, the slope of the line is $-\frac{1}{2}$, so the slope of the perpendicular is 2.

R6-3. Solving we obtain $x \geq \frac{T - 1}{3}$. Since $T = 2$, $x \geq \frac{1}{3}$, so the answer is $\frac{1}{3}$.

This last relay is a very simple one but it does illustrate how these could be used in class for a fun review.

Many teams practice relay races quite a bit since each year the competition comes down to which team can score the most points on the relays. Even when the individual questions seem somewhat easy, the number of points scored on a relay is often surprisingly low. Prior to 2009's contest there were 20 points for each relay and a score of 10 points on a relay was absolutely outstanding. Usually, teams got 2, 4 or 6.

The relays in this collection are easier than the actual relays but I think they do a good job of indicating the kinds of thinking that goes on. The thinking skills required in relays are a little different than those required to do well on textbook problems. Students learn to be more aggressive, to trust their intuition, and to gamble. The problems have a certain amount of ambiguity until one receives an answer and I think that is a valuable situation for any student who wants to be a good problem solver to be in.