2019 ARML Local Contest: Sponsored by Star League

Photocopying Instructions

Make one copy of the whole packet for each team. It contains:

1 copy of the Team Score Sheet
1 copy of the Team Round Answer Sheet
6 copies of the Team Round (2 pages)
1 copy of the Individual Round questions (for proctor)
6 copies of each Individual Round pair
1 copy of the Relay Round Answer Sheets
(Cut out the 12 miniature answer sheets)
1 copy of each Relay Round Sheet (6 pages for each round)
6 copies of the Tiebreaker Question

Notes:
No calculators are allowed for any round.
Make sure copious scratch paper is available.
Thank you so much for coordinating ARML Local.

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Team Score Sheet

Team Name: ______________________________

Team Round (4 pts per correct answer, 60 max.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Tot

Individual Round (1 pt per correct answer, 60 max.)

Student Names	1	2	3	4	5	6	7	8	9	10	Tot
1.											
2.											
3.											
4.											
5.											
6.											
Totals											

Relay Round

(Round 1: 3x 2pts/1pt. Round 2: 2x 4pts/2pts, Round 3: 1x 6pts/3pts. 20 points max.)

Relay Round/Team	1/1	1/2	1/3	2/1	2/2	3	Total
Score							

Total (out of 140): __________

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Team Round Answer Sheet

Team Name: ______________________________

Question Number	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

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Team Round (45 minutes)

T-1 Compute the number of ordered pairs of positive integers (x, y) that satisfy the equality $x^y = 2^{9!}$.

T-2 Let $PQRS$ be a rectangle and let A, B, C, and D be points such that A is on $\overline{PQ}$, D is on $\overline{RS}$, and B and C are inside the rectangle such that $\overline{AB} \parallel \overline{PS}$, $\overline{AB} \perp \overline{BC}$, and $\overline{BC} \perp \overline{CD}$. Point E lies on $\overline{RS}$ such that $\overline{AE}$ intersects $\overline{BC}$ and $[PABCDS] = [PAES]$. Given that $AB = 30$, $BC = 24$, and $CD = 10$, compute DE.

T-3 Given that the numbers $\log_2(\log_2 x)$, $\log_4(\log_4 x)$, and $\log_{16}(\log_{16} x)$ form an arithmetic progression in that order, compute x.

T-4 Compute the maximum value of the function $f(x) = \sin(x) + \sin(x + \cos^{-1} \frac{4}{7})$ as x varies over all real numbers.

T-5 Lulu the Magical Pony has developed a wagering strategy for seven days at the casino. He plays the same game five times each day, but wagers different amounts each day. For $1 \leq N \leq 7$, on day N, for each play of the game he either wins $\$2 \times 5^{N-1}$ or loses $\$5^{N-1}$. At the end of the week, Lulu has a profit of \$20,119. Compute the total number of games that Lulu won over the week.

T-6 Let $\mathcal{T}$ be an isosceles trapezoid such that there exists a point P in the plane of $\mathcal{T}$ whose distances to the four vertices of $\mathcal{T}$ are 5, 6, 8, and 11. Compute the greatest possible ratio of the length of the longer base of $\mathcal{T}$ to the length of its shorter base.

T-7 Let $f(x) = x^2 - 2\sqrt{2}x + 3$. Define the functions $g_n\big(f(x)\big)$ such that $g_1\big(f(x)\big) = f(x)$ and $g_{n+1}\big(f(x)\big) = f(x)^{g_n(f(x))}$ for all $n \geq 1$. Compute the sum of all distinct real values x such that $g_{2019}(f(x)) = 2019$.

T-8 Integers a, b, c, d, e, and f are chosen uniformly at random and with replacement from the set $\{1, 2, \ldots, 12\}$. Compute the probability that $a^b c^d e^f - 1$ is divisible by 3.

T-9 Positive numbers x, y, and z satisfy $x^3y^2z = 36$. Compute the least possible value of $2x + y + 3z$.

T-10 Compute the least positive value of θ in radians for which $\sin\theta + \sin(2\theta) + \cdots + \sin(2019\theta) = 0$.

T-11 In $\triangle ABC$, D is on $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$. Let P and Q be the incenters of $\triangle ABD$ and $\triangle ADC$, respectively. Given that $BC = 14$, $AD = 12$, $CD = 5$, and O is the circumcenter of $\triangle ABC$, compute $[OPQ]$.

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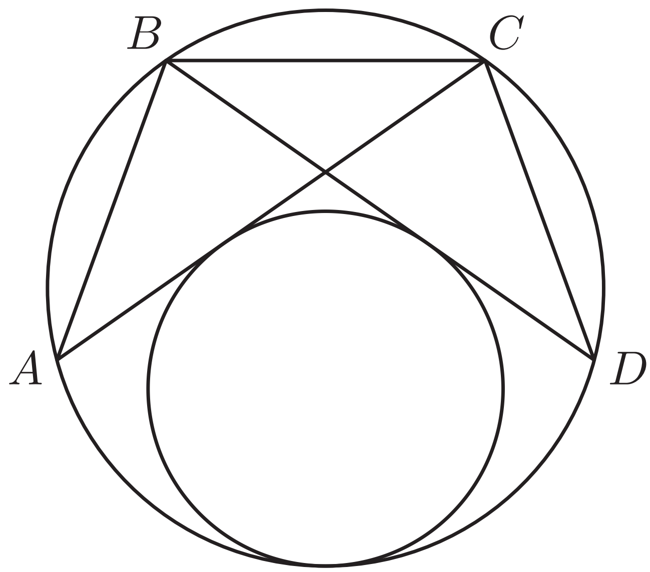

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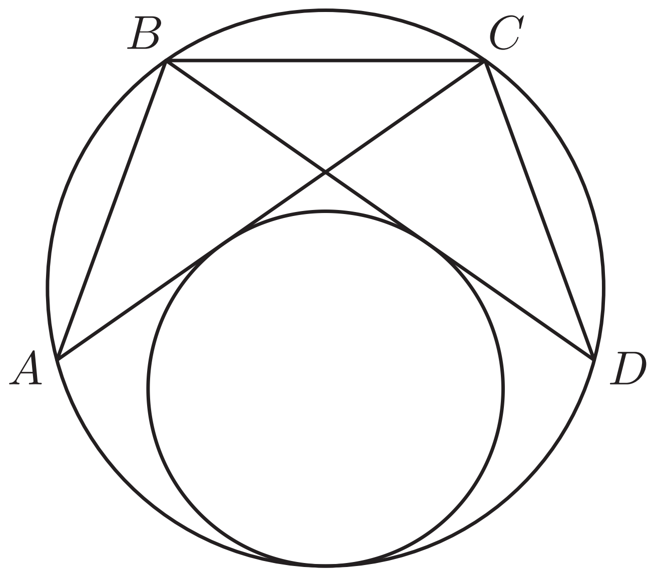

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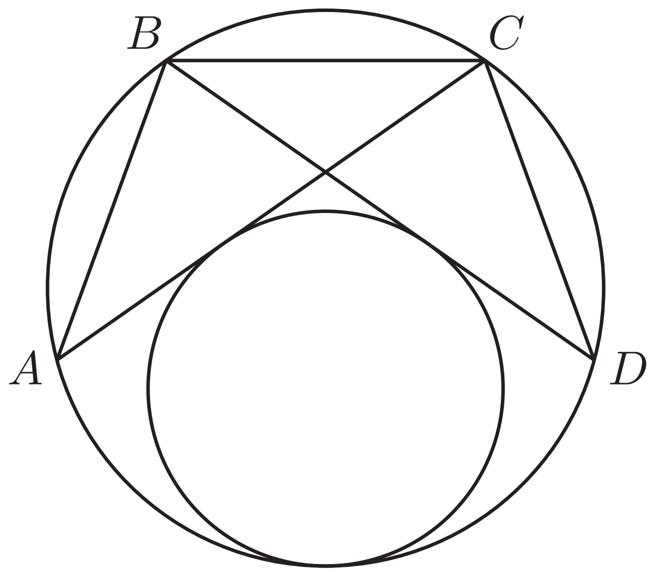

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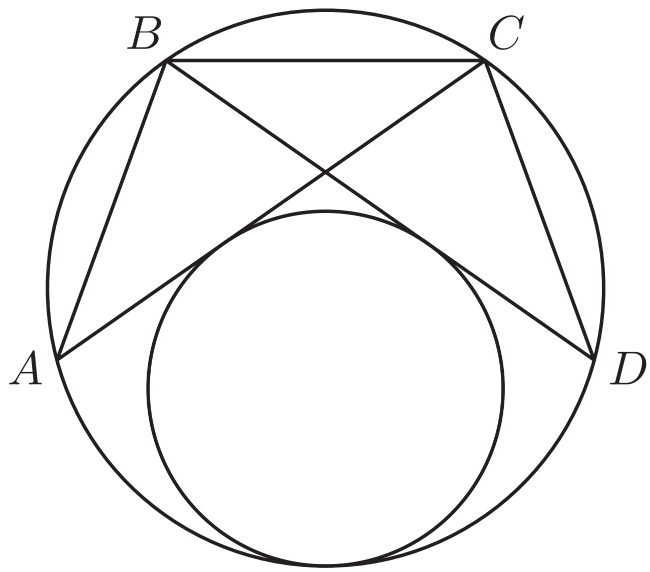

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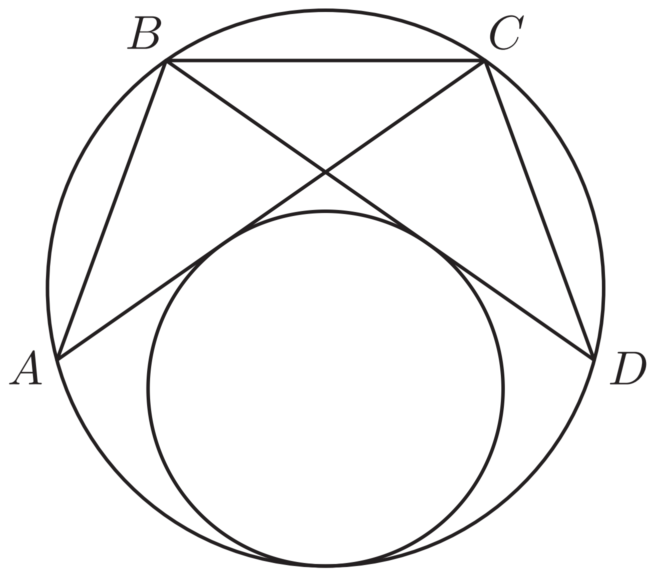

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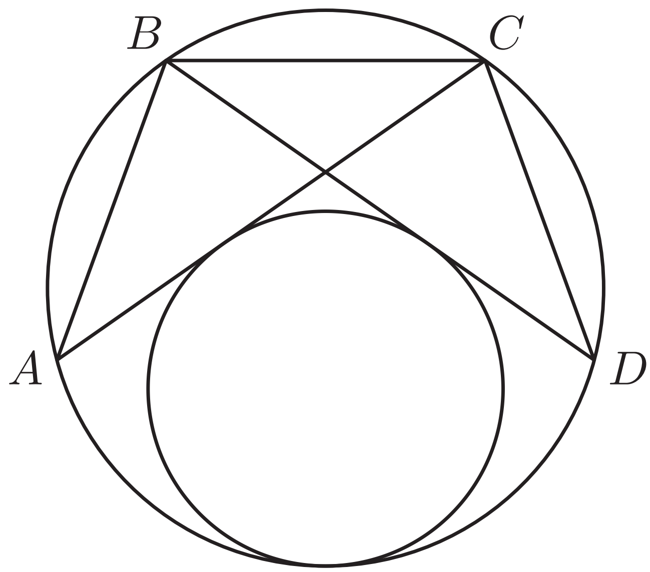

T-15 Given that $\displaystyle\sum_{k=0}^{\infty} \binom{2k}{k} \frac{1}{5^k} = \sqrt{5}$, compute the value of the sum

$$\sum_{k=0}^{\infty} \binom{2k+1}{k} \frac{1}{5^k}.$$

2019 ARML Local: Sponsored by Star League
Copy of Individual Round Questions for Proctor
Individual Round (10 minutes per pair)

I-1 Compute the least positive integer n such that $2n$ and $3n$ both contain the digit 7 when written in base ten.

I-2 Circle $\mathcal{C}$ lies entirely inside rectangle $\mathcal{R}$, and $\mathcal{C}$ is tangent to three of the sides of $\mathcal{R}$. Given that the ratio of the area of $\mathcal{C}$ to the area of $\mathcal{R}$ is $\frac{\pi}{5}$, compute the ratio of the circumference of $\mathcal{C}$ to the perimeter of $\mathcal{R}$.

I-3 Compute the sum of all integers k with $1 \leq k \leq 2019$ such that $(2k-1)^2 = 1 + k \times 10^{k/13}$.

I-4 Exactly five distinct vertices of a regular 12-gon are randomly colored red, with all vertices equally likely to be colored. Compute the probability that no edge of the 12-gon has both its vertices colored red.

I-5 Triangle SAM lies in the coordinate plane with $S = (-15, 0)$, $A = (0, 36)$, and $M = (15, 0)$. There is a unique point $B = (x, y)$ in the interior of $\triangle SAM$ such that the perimeters of $\triangle BSA$, $\triangle BAM$, and $\triangle BMS$ are equal. Compute $x + y$.

I-6 The function $f(x) = x^4 + ax^3 + bx^2 + 2000x + d$ has four distinct roots. Two of the roots sum to 5; the other two roots also sum to 5. Compute b.

I-7 Compute the sum of all possible values for d that satisfy the following property: there exists a positive integer n with exactly 15 divisors such that $2n$ has exactly d divisors.

I-8 Let $\theta = \frac{1}{2}\sin^{-1}\frac{2}{3}$. Compute $\sin^4\theta + \cos^4\theta$.

I-9 Let P be a set of five points in the plane, no three of which are colinear. Let S be the set of all line segments between two points in P. Compute the number of subsets of S such that exactly four triangles have all their edges in S.

I-10 Let m and n be positive integers. When the point $(15, 17)$ is reflected across the line $y = mx$ and then the reflected point is reflected across the line $y = nx$, the resulting point is $(17, 15)$. Compute $m + n$.

2019 ARML Local: Sponsored by Star League
Individual Questions 1 and 2
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-1: [ ]

Answer to I-2: [ ]

I-1: Compute the least positive integer n such that $2n$ and $3n$ both contain the digit 7 when written in base ten.

I-2: Circle $\mathcal{C}$ lies entirely inside rectangle $\mathcal{R}$, and $\mathcal{C}$ is tangent to three of the sides of $\mathcal{R}$. Given that the ratio of the area of $\mathcal{C}$ to the area of $\mathcal{R}$ is $\frac{\pi}{5}$, compute the ratio of the circumference of $\mathcal{C}$ to the perimeter of $\mathcal{R}$.

2019 ARML Local: Sponsored by Star League
Individual Questions 1 and 2
(10 minutes)

Name: ________________________________

Team: ________________________________

Answer to I-1: [ ]

Answer to I-2: [ ]

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2019 ARML Local: Sponsored by Star League
Individual Questions 1 and 2
(10 minutes)

Name: ______________________________

Team: ______________________________

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Answer to I-2: [ ]

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2019 ARML Local: Sponsored by Star League
Individual Questions 1 and 2
(10 minutes)

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Team: ________________________________

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Answer to I-2: [ ]

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2019 ARML Local: Sponsored by Star League
Individual Questions 1 and 2
(10 minutes)

Name: ______________________________

Team: ______________________________

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Answer to I-2:

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2019 ARML Local: Sponsored by Star League
Individual Questions 1 and 2
(10 minutes)

Name: ______________________________

Team: ______________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 3 and 4
(10 minutes)

Name: ________________________________

Team: ________________________________

Answer to I-3: [ ]

Answer to I-4: [ ]

I-3: Compute the sum of all integers k with $1 \leq k \leq 2019$ such that $(2k-1)^2 = 1 + k \times 10^{k/13}$.

I-4: Exactly five distinct vertices of a regular 12-gon are randomly colored red, with all vertices equally likely to be colored. Compute the probability that no edge of the 12-gon has both its vertices colored red.

2019 ARML Local: Sponsored by Star League
Individual Questions 3 and 4
(10 minutes)

Name: ______________________________

Team: ______________________________

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Answer to I-4: [ ]

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2019 ARML Local: Sponsored by Star League
Individual Questions 3 and 4
(10 minutes)

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Team: ________________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 3 and 4
(10 minutes)

Name: ______________________________

Team: ______________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 3 and 4
(10 minutes)

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Team: ________________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 3 and 4
(10 minutes)

Name: ________________________________

Team: ________________________________

Answer to I-3: | |

Answer to I-4: | |

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2019 ARML Local: Sponsored by Star League
Individual Questions 5 and 6
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-5: [ ]

Answer to I-6: [ ]

I-5: Triangle SAM lies in the coordinate plane with $S = (-15, 0)$, $A = (0, 36)$, and $M = (15, 0)$. There is a unique point $B = (x, y)$ in the interior of $\triangle SAM$ such that the perimeters of $\triangle BSA$, $\triangle BAM$, and $\triangle BMS$ are equal. Compute $x + y$.

I-6: The function $f(x) = x^4 + ax^3 + bx^2 + 2000x + d$ has four distinct roots. Two of the roots sum to 5; the other two roots also sum to 5. Compute b.

2019 ARML Local: Sponsored by Star League
Individual Questions 5 and 6
(10 minutes)

Name: ______________________________

Team: ______________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 5 and 6
(10 minutes)

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Team: ________________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 5 and 6
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2019 ARML Local: Sponsored by Star League
Individual Questions 5 and 6
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2019 ARML Local: Sponsored by Star League
Individual Questions 5 and 6
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Team: ______________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 7 and 8
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-7:

Answer to I-8:

I-7: Compute the sum of all possible values for d that satisfy the following property: there exists a positive integer n with exactly 15 divisors such that $2n$ has exactly d divisors.

I-8: Let $\theta = \frac{1}{2}\sin^{-1}\frac{2}{3}$. Compute $\sin^4\theta + \cos^4\theta$.

2019 ARML Local: Sponsored by Star League
Individual Questions 7 and 8
(10 minutes)

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2019 ARML Local: Sponsored by Star League
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2019 ARML Local: Sponsored by Star League

Individual Questions 7 and 8

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2019 ARML Local: Sponsored by Star League
Individual Questions 7 and 8
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2019 ARML Local: Sponsored by Star League
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2019 ARML Local: Sponsored by Star League
Individual Questions 9 and 10
(10 minutes)

Name: ______________________________

Team: ______________________________

Answer to I-9: [ ]

Answer to I-10: [ ]

I-9: Let P be a set of five points in the plane, no three of which are colinear. Let S be the set of all line segments between two points in P. Compute the number of subsets of S such that exactly four triangles have all their edges in S.

I-10: Let m and n be positive integers. When the point $(15, 17)$ is reflected across the line $y = mx$ and then the reflected point is reflected across the line $y = nx$, the resulting point is $(17, 15)$. Compute $m + n$.

2019 ARML Local: Sponsored by Star League
Individual Questions 9 and 10
(10 minutes)

Name: ____________________

Team: ____________________

Answer to I-9:

Answer to I-10:

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2019 ARML Local: Sponsored by Star League
Individual Questions 9 and 10
(10 minutes)

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Team: ________________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 9 and 10
(10 minutes)

Name: ________________________________

Team: ________________________________

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2019 ARML Local: Sponsored by Star League
Individual Questions 9 and 10
(10 minutes)

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2019 ARML Local: Sponsored by Star League
Individual Questions 9 and 10
(10 minutes)

Name: ______________________________

Team: ______________________________

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2019 ARML Local Contest: Sponsored by Star League

Relay Round Answer Sheets

Team Name: **Relay 1, Team 1 Answer (3 minutes, 2 points)**	**Team Name:** **Relay 1, Team 1 Answer (6 minutes, 1 point)**
Team Name: **Relay 1, Team 2 Answer (3 minutes, 2 points)**	**Team Name:** **Relay 1, Team 2 Answer (6 minutes, 1 point)**
Team Name: **Relay 1, Team 3 Answer (3 minutes, 2 points)**	**Team Name:** **Relay 1, Team 3 Answer (6 minutes, 1 point)**
Team Name: **Relay 2, Team 1 Answer (4 minutes, 4 points)**	**Team Name:** **Relay 2, Team 1 Answer (8 minutes, 2 points)**
Team Name: **Relay 2, Team 2 Answer (4 minutes, 4 points)**	**Team Name:** **Relay 2, Team 2 Answer (8 minutes, 2 points)**
Team Name: **Relay 3, Team Answer (5 minutes, 6 points)**	**Team Name:** **Relay 3, Team Answer (10 minutes, 3 points)**

2019 ARML Local: Sponsored by Star League

Relay 1

(6 minutes)

R1-1: Three 1×1 squares in a 3×3 grid are chosen randomly and colored orange. Compute the probability that no row or column of the grid contains more than one orange square.

2019 ARML Local: Sponsored by Star League

Relay 1

(6 minutes)

R1-2: Let $T = \text{TNYWR}$. Compute the sum of the prime factors of $\dfrac{1-T^2}{T^3}$.

2019 ARML Local: Sponsored by Star League
Relay 1
(6 minutes)

R1-1: Three 1×1 squares in a 3×3 grid are chosen randomly and colored orange. Compute the probability that no row or column of the grid contains more than one orange square.

2019 ARML Local: Sponsored by Star League
Relay 1
(6 minutes)

R1-2: Let $T = \text{TNYWR}$. Compute the sum of the prime factors of $\dfrac{1-T^2}{T^3}$.

2019 ARML Local: Sponsored by Star League
Relay 1
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2019 ARML Local: Sponsored by Star League
Relay 1
(6 minutes)

R1-2: Let $T = \text{TNYWR}$. Compute the sum of the prime factors of $\dfrac{1-T^2}{T^3}$.

2019 ARML Local: Sponsored by Star League

Relay 2

(8 minutes)

R2-1: Let $a_1, a_2, a_3, \ldots$ be a nonconstant arithmetic sequence. Suppose that a_1, a_{11}, a_{111} is a geometric sequence. Compute a_{11}/a_1.

2019 ARML Local: Sponsored by Star League
Relay 2
(8 minutes)

R2-2: Let $T = \text{TNYWR}$. Frank has a $3 \times T$ grid of squares. Compute the number of ways that Frank can shade some (possibly none) of the squares in the grid, such that each row and column contains at most one shaded square.

2019 ARML Local: Sponsored by Star League
Relay 2
(8 minutes)

R2-3: Let $T =$ TNYWR. Let N be the positive integer created by joining T copies of T. (For example, if $T = 15$, then N is the 30-digit integer $151515\ldots15$.) Compute the remainder when N is divided by 99.

2019 ARML Local: Sponsored by Star League

Relay 2

(8 minutes)

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2019 ARML Local: Sponsored by Star League
Relay 2
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2019 ARML Local: Sponsored by Star League

Relay 3

(10 minutes)

R3-1: Given that $(20^{19}+100)^2-(20^{19}-100)^2=20^N$, compute N.

2019 ARML Local: Sponsored by Star League

Relay 3

(10 minutes)

R3-2: Let $T = \text{TNYWR}$. Given that a rectangle has area T and perimeter $T - 1$, compute the length of the shorter side of the rectangle.

2019 ARML Local: Sponsored by Star League
Relay 3
(10 minutes)

R3-3: Let T = TNYWR. Let N be the sum of all positive integers which divide $2^7(T^8 - 1)$. Compute the remainder when N is divided by 100.

2019 ARML Local: Sponsored by Star League

Relay 3

(10 minutes)

R3-4: Let T = TNYWR. Fran flips four fair coins and rolls a standard, fair six-sided die. Let p be the probability that the number rolled on the die is equal to the number of heads flipped. Compute pT.

2019 ARML Local: Sponsored by Star League

Relay 3

(10 minutes)

R3-5: Let $T = \text{TNYWR}$. In $\triangle ABC$, point M is the midpoint of $\overline{BC}$ and D is the foot of the altitude from A to $\overline{BC}$, with D between B and M. Given that $AC = T$, $AM = 4$, and $BD = \frac{3}{4}$, compute AB.

2019 ARML Local: Sponsored by Star League

Relay 3

(10 minutes)

R3-6: Let $T = $ TNYWR. For a positive integer n, let S_n be the sum of the first n positive integers. Compute the remainder of S_{T^4} when divided by 9.

2019 ARML Local: Sponsored by Star League
Tiebreaker
(10 minutes)

Name: ________________________________

Team: ________________________________

Time to submit answer (seconds): ________________________________

Answer to Tiebreaker:

Compute the number of orderings of the numbers $1, 2, \ldots, 9$ with the following property: if m comes before n in the ordering, then $m < n^2$.

2019 ARML Local: Sponsored by Star League
Tiebreaker
(10 minutes)

Name: ______________________________

Team: ______________________________

Time to submit answer (seconds): ______________________________

Answer to Tiebreaker: [ ]

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2019 ARML Local: Sponsored by Star League

Tiebreaker

(10 minutes)

Name: ______________________________

Team: ______________________________

Time to submit answer (seconds): ______________________________

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2019 ARML Local: Sponsored by Star League
Tiebreaker
(10 minutes)

Name: ______________________________

Team: ______________________________

Time to submit answer (seconds): ______________________________

Answer to Tiebreaker: | |

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2019 ARML Local: Sponsored by Star League
Tiebreaker
(10 minutes)

Name: ______________________________

Team: ______________________________

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2019 ARML Local: Sponsored by Star League
Tiebreaker
(10 minutes)

Name: ______________________________

Team: ______________________________

Time to submit answer (seconds): ______________________________

Answer to Tiebreaker: [ ]

Compute the number of orderings of the numbers $1, 2, \ldots, 9$ with the following property: if m comes before n in the ordering, then $m < n^2$.

2019 ARML Local Contest: Sponsored by Star League

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If there are any questions about the contest, please contact the ARML Local Head Coordinator ASAP.

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