Some Mathematical Ideas Used in the Competition
Revised February 2014

1. The word *compute* will always call for an exact answer in simplest form. Thus final answers such as

$$\frac{6}{4}, 5+2, 2^5, 2\sin 30°, \sqrt{12}, \frac{4}{\sqrt{2}}, \text{ and } \frac{5}{1+2i}$$

are not satisfactory; they should instead be expressed as

$$\frac{3}{2}, 7, 32, 1, 2\sqrt{3}, 2\sqrt{2}, \text{ and } 1-2i,$$

respectively. In cases where there is a question as to what is *most simplified*, alternative answers may be accepted. For example, $\frac{3}{2}, 1\frac{1}{2}$, and 1.5 are all acceptable unless the problem specifies a certain form. The judge's decision is final. In the Power Question, *compute* means that a numerical answer must be supplied but a proof is not necessary.

2. When an answer is called for as an *ordered pair* (a, b), it must given in precisely that form, including the parentheses and the comma. The same applies for other choices of letters and for ordered n-tuples.

3. The length of the sides opposite vertices A, B, and C of triangle ABC will be represented by the lowercase letters a, b, and c, respectively. Depending on the context, the letter A may represent the vertex, the angle, or the measure of the angle. A similar convention holds for other choices of letters representing a triangle. If a quadrilateral is named $ARML$, it is understood that the vertices A, R, M, and L occur in this order around the polygon, either clockwise or counterclockwise. This convention holds for other choices of letters and for the naming of polygons in general. Unless otherwise specified, references to polygons, including triangles, should be understood to mean simple, non-degenerate ones.

4. If $A_1, A_2, \ldots, A_n$ are the vertices of a polygon in order, then $[A_1 A_2 \ldots A_n]$ denotes the area enclosed by the polygon.

5. If a diagram is given with a problem, it is not necessarily drawn to scale.

6. The *Floor Function*, also called the greatest integer function, is denoted by $\lfloor x \rfloor$. It is defined as follows: $\lfloor x \rfloor = n$, where n is the unique integer such that $n \le x < n+1$. The *Ceiling Function*, also called the least integer function, is denoted by $\lceil x \rceil$. It is defined as follows: $\lceil x \rceil = n$, where n is the unique integer such that $n-1 < x \le n$.

7. The *fractional part* of a real number x is denoted $\{x\}$, and is defined by $\{x\} = x - \lfloor x \rfloor$.

8. For sets, $\max\{a_1, a_2, \ldots, a_n\}$ denotes the largest element of the set and $\min\{a_1, a_2, \ldots, a_n\}$ denotes the least element of the set.

9. The *greatest lower bound* of a set is the largest number that is less than or equal to all the elements of the set, if such a number exists. Thus 2 is the greatest lower bound for both $\{x : 2 < x\}$ and $\{x : 2 \le x\}$. The *least upper bound* of a set is the smallest number that is greater than or equal to all elements of the set, if such a number exists. Thus 3 is the least upper bound for both $\{x : x < 3\}$ and $\{x : x \le 3\}$.

10. Logarithms are base 10 unless otherwise indicated. Logarithms are only defined for positive real number arguments. In logarithm expressions, or when numbers are written in a base other than 10, the base will usually be written as a subscript. For example, the expression $\log_4 64$ refers to the logarithm base 4 of 64 (and equals 3), while the numeral 321_4 is in base 4 (and equals $3 \cdot 4^2 + 2 \cdot 4^1 + 1 \cdot 4^0 = 57$).

11. If *complex numbers* are used, i will stand for $\sqrt{-1}$.

12. Symbols for *combinations* and *permutations*: The number of combinations of n objects taken r at a time is denoted $\binom{n}{r}$ or ${}_nC_r$. The number of permutations of n objects taken r at a time is denoted ${}_nP_r$. These functions are given by the formulae

$$\binom{n}{r} = {}_nC_r = \frac{n!}{(n-r)! \cdot r!};$$
$$\phantom{\binom{n}{r} =} {}_nP_r = \frac{n!}{(n-r)!}.$$

13. Unless otherwise specified, in any problem using trigonometry, all trigonometric expressions are given in radians. The expressions Arcsin x, $\sin^{-1} x$, Arccos x, $\cos^{-1} x$, Arctan x, and $\tan^{-1} x$ refer to the principal values of these inverse trigonometric functions. Their ranges are as follows:

$$-\frac{\pi}{2} \leq \quad \sin^{-1} x \quad \leq \frac{\pi}{2}$$
$$0 \leq \quad \cos^{-1} x \quad \leq \pi$$
$$-\frac{\pi}{2} < \quad \tan^{-1} x \quad < \frac{\pi}{2}.$$

14. *Lattice points* are points all of whose coordinates are integers. For example, the points $(3, 0, -2)$ and $(5, 5)$ are lattice points; the point $(3, \frac{3}{2})$ is not a lattice point.

15. If a problem refers to the digits of a number, those digits are usually underlined to distinguish the digits of a number from the product of those integers. For example: "Find the missing digits A and B if $K = \underline{A}\,2\,5\,\underline{B}$ and K is a multiple of 72." In this case, K is **not** the product $A \cdot 2 \cdot 5 \cdot B$. Unless otherwise specified, numerals are written in base 10, and problems about digits refer to the digits in base 10. A *palindrome* is a number, not ending in the digit 0, whose digits read the same backwards and forwards, for example 171, 1771, and 5555. In bases other than ten, digits with values greater than 9 will be understood to be represented by single symbols. In general, those digits will be written as base-ten numerals in parentheses, for example, in base 289, the two-digit number $(10)(15)_{289}$ represents $10 \cdot 289 + 15 = 2905_{10}$. In special cases, particularly hexadecimal (base 16), these values are often represented by the letters $A = 10$, $B = 11$, $C = 12$, $D = 13$, $E = 14$, and $F = 15$. For example, the two-digit hexadecimal numeral $B3$ represents $11 \cdot 16 + 3 = 179_{10}$. Because a palindrome has the same sequence of digits read forwards and backwards, $(17)5(17)$ is a three-digit palindrome in (for example) base 20, but $(17)71$ is not.

16. *Divisors* or *factors* of an integer refer to positive integers only. *Proper divisors* of an integer are those divisors that are less than the integer itself, and include 1.

17. The designation *primes* refers to positive primes only. Note: 1 is a unit, not a prime.